Homework 2

Due Wednesday January 28, 2009 at the beginning of class.

Please write your home work neatly and use dark ink or a dark pencil. Please write in complete sentences. If you do something clever, you might want to give me a hint about what you are doing. If you want to replace some of the problems I have assigned with something more interesting, that is fine with me.

(7) Page 9, problem 1.9. (We have already used this result many times. I meant to write it down in class, but I never got around to it.)

(8) Page 9, problem 1.7. (I don't know how I missed this problem when I assigned the first set of problems.)

(9) Let n be an arbitrary positive integer (for example 4). Let R be the polynomial ring $k[\{a_{i,j} \mid 1 \le i \le j \le n\}]$, A be the "generic symmetric $n \times n$ matrix":

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,n} \\ a_{1,2} & a_{2,2} & \dots & a_{2,n} \\ \vdots & \vdots & & \vdots \\ a_{1,n} & a_{2,n} & \dots & a_{n,n} \end{bmatrix},$$

and $V \subseteq R$ be the vector space which is spanned by the 2×2 minors of A. Find the dimension of V. (At first glance, this appears to be a Combinatorics problem. However, I am not able to solve the problem using Combinatorics alone. I can solve the problem using Commutative Algebra. There is a huge branch of Combinatorics which uses Commutative Algebra or Algebraic Geometry to solve Combinatorics problems. Stanley (MIT) is probably the founder of this school of thought which also includes Björner (Stockholm), Garsia (San Diego), Haiman (Berkeley) and many more folks.)