

Exercise 4.14*. Use the hook length formula to show that the only irreducible representations of \mathfrak{S}_d of dimension less than d are the trivial and alternating representations U and U' of dimension 1, the standard representation V and $V' = V \otimes U'$ of dimension $d - 1$, and three other examples: the two-dimensional representation of \mathfrak{S}_4 corresponding to the partition $4 = 2 + 2$, and the two five-dimensional representations of \mathfrak{S}_6 corresponding to the partitions $6 = 3 + 3$ and $6 = 2 + 2 + 2$.

Exercise 4.15*. Using Frobenius's formula or otherwise, show that:

$$\chi_{(d-1,1)}(C_i) = i_1 - 1;$$

$$\chi_{(d-2,1,1)}(C_i) = \frac{1}{2}(i_1 - 1)(i_1 - 2) - i_2;$$

$$\chi_{(d-2,2)}(C_i) = \frac{1}{2}(i_1 - 1)(i_1 - 2) + i_2 - 1.$$

Can you continue this list?