Homework 1

Due Friday January 18, 2008 at the beginning of class.

Let V be a vector space over the field K and let $\operatorname{Sym}_m V$ be the m^{th} symmetric power of V. It is clear that $\operatorname{Sym}_m V$ is a representation of $\mathrm{GL}(V)$. When is $\operatorname{Sym}_m V$ an irreducible representation of $\mathrm{GL}(V)$? Prove your answer. (Feel free to start with the assumptions that K is the field of complex numbers, $\dim_K V = 2$ and $m = 2$. Feel free also to drop as many of these hypotheses as you can.)

If you are not familiar with Symmetric Modules (and I do not assume that you are), then you may look at the problem this way: Let $v_1, \ldots, v_n$ be a basis for V over K. Take $\operatorname{Sym}_m(V)$ to be the vector space of all homogeneous polynomials of degree m in the variables $v_1, \ldots, v_n$.