

MATH 702 – SPRING 2024
HOMEWORK 1
DUE MONDAY, JANUARY 29, 2024 BY THE BEGINNING OF CLASS.

1. Prove that $\mathbb{Z}[\sqrt{-5}]$ is not a UFD.
2. Express the ideal (2) in the ring $\mathbb{Z}[\sqrt{-5}]$ as the product of prime ideals. (If I and J are ideals of the (commutative) ring R , then IJ is the smallest ideal of R that contains all elements of the form ij with $i \in I$ and $j \in J$.)
3. Find a commutative domain R and an element r in R with r not 0, r not a unit, and r not equal to a finite product of irreducible elements of R .
4. Prove that $\mathbb{Z}[i]$ is a Euclidean domain. (Let R be a domain. Suppose that there is a function f from $R \setminus \{0\}$ to the set of non-negative integers with the property that whenever a and b are elements of R with b not zero, then there exists q and r in R such that $a = bq + r$ and either $r = 0$ or $f(r) < f(b)$. In this case R is called a Euclidean Domain.)