

**MATH 702 – SPRING 2024**  
**EXAM 2**

**Please e-mail your typed answer to me before class on Monday, April 1.**

Let  $\zeta = e^{2\pi i/17}$ . Exhibit complex numbers  $u_1, \dots, u_n$ , for some  $n$ , such that

$$u_i^2 \in \mathbb{Q}[u_1, \dots, u_{i-1}],$$

for  $1 \leq i \leq n$ , and  $\zeta \in \mathbb{Q}[u_1, \dots, u_n]$ . Please prove your assertions.

- Remarks.**
1. Please do this on your own as much as you can. If you have to consult things that are not already in your head, try to keep your consultation to be as small as possible.
  2. I hope for a Galois Theory argument (rather than a whole bunch of Trigonometry formulas that neither of us know).
  3. Of course, by answering this question you are giving a procedure for constructing a regular 17-gon by ruler and compass.