MATH 701 – FALL 2023
HOMEWORK 6
DUE MONDAY, NOVEMBER 6 BY THE BEGINNING OF CLASS.

13. Let G be a group of order p^n for some prime p and let H be a normal subgroup of G, with $H \neq \{\mathrm{id}\}$. Prove that $Z(G) \cap H \neq \{\mathrm{id}\}$, where $Z(G)$ is the center of G.
14. How many elements of order 7 are there in a simple group of order 168?
15. Classify all groups of order $2p$ where p is an odd prime integer. (This instruction means state and prove a result which says, "If G is a group of order $2p$, where p is an odd prime integer, then G is isomorphic to exactly one of the following groups: … .")
16. Let G be a group of order 30. Prove that G has a subgroup of order 15.