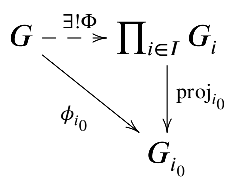
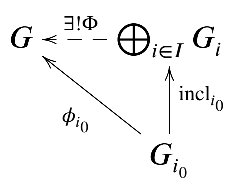
I changed my mind. I decided that it is not reasonable to make you turn in these problems before the exam.

MATH 701 – FALL 2023
HOMEWORK 5
DUE MONDAY, OCTOBER 30 BY THE BEGINNING OF CLASS.

9. (True or False) If true, prove it. If false, give a counterexample. Let G be a group. If x and y are elements of G of finite order, then xy has finite order.
10. (True or False) If true, prove it. If false, give a counterexample. Let G be a group. If x and y are elements of G of finite order and $xy = yx$, then the order of xy is equal to the least common multiple of the order of x and the order of y.
11. Let I be an index set. Suppose that for each $i \in I$, G_i is a group. Consider the direct product $\prod_{i\in I} G_i$. For each $i_0 \in I$, let $\text{proj}_{i_0} : \prod_{i\in I} G_i \to G_{i_0}$ be the natural projection map. Let G be a group and, for each i, let $\phi_i : G \to G_i$ be a group homomorphism. Prove that there exists a unique group homomorphism $\Phi : G \to \prod_{i\in I} G_i$ so that the diagram



commutes for all $i_0 \in I$.
12. Let I be an index set. Suppose that for each $i \in I$, G_i is a group. Consider the direct sum $\bigoplus_{i\in I} G_i$. For each $i_0 \in I$, let $\text{incl}_{i_0} : G_{i_0} \to \bigoplus_{i\in I} G_i$ be the natural inclusion map. Let G be an Abelian group and, for each i, let $\phi_i : G_i \to G$ be a group homomorphism. Prove that there exists a unique group homomorphism $\Phi : \bigoplus_{i\in I} G_i \to G$ so that the diagram



commutes for all $i_0 \in I$.

Remark. In case it is not already obvious, if i_0 is an element of the index set I, then

$$\text{proj}_{i_0} : \prod_{i\in I} G_i \to G_{i_0}$$

is the projection map which sends the arbitrary element $(g_i)_{i\in I}$ of $\prod_{i\in I} G_i$ to g_{i_0} and

$$\text{incl}_{i_0} : G_{i_0} \to \bigoplus_{i\in I} G_i$$

is the inclusion map which sends the arbitrary element g_{i_0} of G_{i_0} to the element $(h_i)_{i\in I}$ of $\bigoplus_{i\in I} G_i$, where

$$h_i = \begin{cases} \text{the identity element of } G_i, & \text{if } i \neq i_0, \text{ and} \\ g_{i_0}, & \text{if } i = i_0. \end{cases}$$