MATH 701 – FALL 2023
HOMEWORK 2
DUE MONDAY, SEPTEMBER 18 BY THE BEGINNING OF CLASS.

3. Let n be a fixed positive integer, and let $\mathbb{C}^*$ be the group $\mathbb{C} \setminus \{0\}$ under multiplication. How many subgroups of $\mathbb{C}^*$ have n elements? What are they? Justify your answer.
4. Let C_2 be the subgroup $\{1, -1\}$ of $\mathbb{C}^*$. Consider the group $G = C_2 \times C_2 \times C_2 \times C_2$, which is the direct product of four copies of C_2. How many four element subgroups does G have? Justify your answer.
5. Prove that every element of $\mathrm{SO}_n(\mathbb{R})$ is diagonalizable over $\mathbb{C}$. (It might make sense to prove a more general statement. An element M of $\mathrm{GL}_n(\mathbb{C})$ is called <u>unitary</u> if $\overline{M}^{\mathrm{T}} M$ is the identity matrix. Prove that every unitary matrix from $\mathrm{GL}_n(\mathbb{C})$ is diagonalizable. In this problem $\overline{}$ means complex conjugate. Recall that $\mathrm{SO}_n(\mathbb{R})$ is the subgroup of $\mathrm{GL}_n(\mathbb{R})$ which consists of all matrices M with $M^{\mathrm{T}} M$ equal to the identity matrix.)
6. Let ℓ be the line in 3-space through the origin and parallel to the vector $\vec{\boldsymbol{i}} + 2\vec{\boldsymbol{j}} + 3\vec{\boldsymbol{k}}$. Let $f : \mathbb{R}^3 \to \mathbb{R}^3$ be rotation by $\pi/4$ radians where ℓ is the axis of revolution. Find a matrix M so that
$$f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = M \begin{bmatrix} x \\ y \\ z \end{bmatrix}.$$
There are two correct answers.