MATH 700
HOMEWORK 2

Due Friday, September 6, 1991 at the beginning of class

1. (Hoffman and Kunze, page 55, number 6.) Let V be the vector space over the complex numbers of all functions from $\mathbb{R}$ to $\mathbb{C}$. Let $f_1(x) = 1$, $f_2(x) = e^{ix}$, and $f_3(x) = e^{-ix}$.
   (a) Prove that f_1, f_2, f_3 are linearly independent.
   (b) Let $g_1(x) = 1$, $g_2(x) = \cos x$, and $g_3(x) = \sin x$. Find an invertible matrix P such that

$$[\,f_1 \quad f_2 \quad f_3\,]\,P = [\,g_1 \quad g_2 \quad g_3\,]\,.$$

2. Let V be a vector space of arbitrary dimension over the field F; let B be a basis for V; and let S be a linearly independent subset of V. Prove that there exists a subset S_1 of B such that $S \cup S_1$ is a basis for V.
3. Let V be the vector space of all polynomials in the variables $X_1, \ldots, X_n$ over the field F.
   a. What is the dimension of the subspace W of V, which consists of all homogeneous polynomials of degree d?
   b. What is the dimension of the subspace W' of V, which consists of all polynomials of degree at most d?