

Exam 2, 1991

Each problem is worth 25 points.

1. Let A be a matrix with characteristic polynomial $(x + 1)^4(x - 2)^2$. How many possible Jordan Canonical Forms are there for A ? Describe them.
2. TRUE OR FALSE. (If the statement is true, then prove it. If the statement is false, then give a counterexample.) Let A and B be matrices over the field of complex numbers. Suppose that 1 is the only eigenvalue of A and 1 is the only eigenvalue of B . If A and B have the same minimal polynomials, the same characteristic polynomials, and their eigenspaces have the same dimension, then A and B are similar.
3. Let V be a vector space over the field F , let I be an infinite index set, and for each $i \in I$, let V_i be a vector space over F . For one

$$W_1 \in \left\{ \text{Hom} \left(\bigoplus_{i \in I} V_i, V \right), \text{Hom} \left(\prod_{i \in I} V_i, V \right) \right\}$$

and one

$$W_2 \in \left\{ \bigoplus_{i \in I} \text{Hom} (V_i, V), \prod_{i \in I} \text{Hom} (V_i, V) \right\},$$

there is an isomorphism $T: W_1 \rightarrow W_2$.

- (a) What is W_1 ?
 - (b) What is W_2 ?
 - (c) Define T .
 - (d) Prove that T is one-to-one.
4. Let $T: V \rightarrow V$ be a linear transformation on a finite dimensional vector space over \mathbb{C} . Then V has only finitely many T -invariant subspaces if and only if

 - (a) Fill in the blank with some statement about the minimal and characteristic polynomials of T .
 - (b) Prove your answer to (a).