

**MATH 700**  
**HOMEWORK 4**

Due Friday, September 20, 1991 at the beginning of class.

1. True or False. If true, prove it; if false, give a counterexample. If  $W_1$ ,  $W_2$ , and  $W_3$  are subspaces of the finite dimensional vector space  $V$ , then

$$\begin{aligned} \dim(W_1 + W_2 + W_3) &= \dim W_1 + \dim W_2 + \dim W_3 - \dim(W_1 \cap W_2) \\ &\quad - \dim(W_1 \cap W_3) - \dim(W_2 \cap W_3) + \dim(W_1 \cap W_2 \cap W_3). \end{aligned}$$

2. Give an example of a non-zero vector space  $V$  and a linear transformation  $T: V \rightarrow V$  with the property that the null space of  $T$  is equal to the image of  $T$ .
3. Let  $T: V \rightarrow V$  be a linear transformation which is not the zero transformation and which is not an isomorphism.
  - (a) If  $\dim V < \infty$ , then prove that there exists a linear transformation  $S: V \rightarrow V$  such that  $ST = 0$ , but  $TS \neq 0$ .
  - (b) Does (a) remain true if the hypothesis  $\dim V < \infty$  is removed? (Prove it or give a counterexample.)