

**MATH 700**  
**HOMEWORK 9**  
**DUE FRIDAY OCTOBER 27, 1989**  
**AT THE BEGINNING OF CLASS.**

1. Let  $M$  and  $N$  be  $n \times n$  matrices over  $\mathbb{R}$ . Suppose that  $M$  and  $N$  are similar over  $\mathbb{C}$ . Prove that  $M$  and  $N$  are similar over  $\mathbb{R}$ .
2. Find a matrix whose minimal polynomial is  $(x + 1)^2(x + 2)^2$ , whose characteristic polynomial is  $(x + 1)^4(x + 2)^3$ , and whose rank is 4.
3. Suppose  $T$  is a linear transformation on  $\mathbb{R}^5$  represented in some basis by a diagonal matrix with entries 1, 1, 3, 3, 3 on the main diagonal. What is the rational canonical form for  $T$ ?
4. Suppose that  $V$  is an  $n$ -dimensional vector space and  $T$  is a linear transformation on  $V$  of rank one. Prove that  $T$  is nilpotent or diagonalizable.