MATH 700

HOMEWORK 6

DUE FRIDAY OCTOBER 6, 1989

AT THE BEGINNING OF CLASS

1. (Brown. page 57, number 9.) Suppose that $\omega\colon V \times V \to F$ is a nondegenerate, bilinear form on a finite dimensional vector space V. Let W be a subspace of V. Set
$$W^{\perp} = \{\alpha \in V \mid \omega(\alpha, \beta) = 0 \text{ for all } \beta \in W\}.$$
Show that $V = W \oplus W^{\perp}$.
2. (Brown. page 57, number 10.) With the same hypotheses as in problem 1, suppose $f \in V^*$. Prove that there exists an element $\alpha \in V$ such that $f(b) = \omega(\alpha, \beta)$ for all $\beta \in V$.
3. (Brown. page 57, number 17.) Let $V = \{p(X) \in \mathbb{R}[X] \mid \deg(p) \leq 5\}$. Suppose that $\omega\colon V \times V \to \mathbb{R}$ is given by $\omega(f, g) = \int_0^1 f(x)g(x)dx$. Find an ω-orthonormal basis of V.
4. (Brown. page 82, number 6.) Let V and W be finite-dimensional vector spaces over the field F. Let K be a field containig F. Prove that
$$\mathrm{Hom}_F(V, W) \otimes_F K \quad \text{and} \quad \mathrm{Hom}_K(V \otimes_F K, W \otimes_F K)$$
are isomorphic as K vector spaces.
5. Let R be a commutative ring. Suppose that M is a maximal ideal of R. Prove that R/M is a field. (If you don't like this problem, then do page 104, number 20, instead.)