MATH 700
HOMEWORK 6

Due Friday, October 17, 2003 at the beginning of class.

The vector spaces in the assignment have arbitrary dimension. All direct sums are internal.

1. Let V_1 and V_2 be subspaces of the vector space V. Suppose that $V_1 + V_2 = V$. Prove that there exists a subspace W of V such that $W \subseteq V_2$ and $V = V_1 \oplus W$.

2. (Answer each question with a COMPLETE proof or a counterexample.) Let X, Y, and Z be subspaces of the vector space V with $X \oplus Y = V$ and $X \oplus Z = V$. Is $Y = Z$? Is $Y \cong Z$?

3. Suppose that $T \colon V \to V$ is a linear transformation on the vector space V which satisfies $TT = T$. Prove that $V = \ker T \oplus \operatorname{im} T$.