

MATH 700
HOMEWORK 4

Due Friday, September 19, 2003 at the beginning of class

1. True or False. If true, prove it; if false, give a counterexample. If W_1 , W_2 , and W_3 are subspaces of the finite dimensional vector space V , then

$$\begin{aligned} \dim(W_1 + W_2 + W_3) &= \dim W_1 + \dim W_2 + \dim W_3 - \dim(W_1 \cap W_2) \\ &\quad - \dim(W_1 \cap W_3) - \dim(W_2 \cap W_3) + \dim(W_1 \cap W_2 \cap W_3). \end{aligned}$$

2. Give an example of a non-zero vector space V and a linear transformation $T: V \rightarrow V$ with the property that the null space of T is equal to the image of T .
3. Let $T: V \rightarrow V$ be a linear transformation which is not the zero transformation and which is not an isomorphism.
- (a) If $\dim V < \infty$, then prove that there exists a linear transformation $S: V \rightarrow V$ such that $ST = 0$, but $TS \neq 0$.
- (b) Does (a) remain true if the hypothesis $\dim V < \infty$ is removed? (Prove it or give a counterexample.)