

MATH 700
HOMEWORK 2

Due Friday, September 5, 2003 at the beginning of class

1. (Hoffman and Kunze, page 55, number 6.) Let V be the vector space over the complex numbers of all functions from \mathbb{R} to \mathbb{C} . Let $f_1(x) = 1$, $f_2(x) = e^{ix}$, and $f_3(x) = e^{-ix}$.
 - (a) Prove that f_1, f_2, f_3 are linearly independent.
 - (b) Let $g_1(x) = 1$, $g_2(x) = \cos x$, and $g_3(x) = \sin x$. Find an invertible matrix P such that

$$[f_1 \ f_2 \ f_3]P = [g_1 \ g_2 \ g_3].$$

2. Let V be a vector space of arbitrary dimension over the field F ; let B be a basis for V ; and let S be a linearly independent subset of V . Prove that there exists a subset S_1 of B such that $S \cup S_1$ is a basis for V .
3. Let V be the vector space of all polynomials in the variables X_1, \dots, X_n over the field F .
 - a. What is the dimension of the subspace W of V , which consists of all homogeneous polynomials of degree d ?
 - b. What is the dimension of the subspace W' of V , which consists of all polynomials of degree at most d ?