

MATH 700
HOMEWORK 1

Due Friday, August 29, 2003 at the beginning of class

1. (Hoffman and Kunze, page 49, number 7b.) Let V be the vector space of all 2×2 matrices over the field F . Let W_1 be the subspace of matrices of the form

$$\begin{bmatrix} x & -x \\ y & z \end{bmatrix}$$

and W_2 be the subspace of matrices of the form

$$\begin{bmatrix} a & b \\ -a & c \end{bmatrix}.$$

Find the dimensions of W_1 , W_2 , $W_1 + W_2$, and $W_1 \cap W_2$.

2. (Hoffman and Kunze, page 49, number 8.) Let V be the vector space of all 2×2 matrices over the field F . Find a basis A_1, A_2, A_3, A_4 for V such that $A_j^2 = A_j$ for each j .
3. (Hoffman and Kunze, page 49, number 14.) Let V be the set of real numbers. Regard V as a vector space over the field of **rational** numbers. Prove that this vector space is not finite-dimensional.
4. Let V be the the vector space of rational functions in one variable over the field of complex numbers. (A typical element of V has the form $\frac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are polynomials with complex coefficients in one variable.) Prove that the dimension of V over \mathbb{C} is not countable.