Math 574, Exam 1, Summer 2007

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you.

Please leave room in the upper left corner for the staple.

There are 8 problems **ON TWO SIDES!**. The exam is worth a total of 50 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

I will post the solutions on my website sometime after 3:15 today.

1. (7 points) Suppose that $A_1, \dots, A_n$ are sets where $n \geq 2$. Suppose also that for all pairs of integers i and j with $1 \leq i < j \leq n$, either $A_i \subseteq A_j$ or $A_j \subseteq A_i$. Prove that there exists an integer i, with $1 \leq i \leq n$, such that $A_i \subseteq A_j$ for all j with $1 \leq j \leq n$.

2. (7 points) Prove that if n is a positive integer, then 133 divides $11^{n+1}+12^{2n-1}$.

3. (6 points) Prove that $1 \cdot 1! + 2 \cdot 2! + \cdots + n \cdot n! = (n+1)! - 1$, whenever n is a positive integer.

4. (6 points) Let f be a function from the real numbers to the real numbers, and let a be a real number. What is the negation of the statement: "For all real numbers $\varepsilon > 0$, there exists a real number $\delta > 0$, such that if x is a real number, with $0 < |x-a| < \delta$, then $|f(x) - f(a)| < \varepsilon$"?

5. (6 points) Let A, B, and C be sets, and $g\colon A \to B$ and $f\colon B \to C$ be functions. Suppose that f is onto and $f \circ g$ is onto. Does g have to be onto? If yes, prove your answer. If no, give a counterexample.

6. (6 points) List the elements of $\mathfrak{P}(\mathfrak{P}(\emptyset))$. (In this problem, if S is a set, then $\mathfrak{P}(S)$ is the power set of S.)

7. (6 points) Let $A_i = \{\dots, -2, -1, 0, 1, \dots, i\}$. Find

   (a) $\bigcup_{i=1}^{n} A_i$, and

   (b) $\bigcap_{i=1}^{n} A_i$.

8. (6 points) Consider the statement "if $3 < x$, then $9 < x^2$ ".
   (a) What is the converse of the original statement?
   (b) Is (a) logically equivalent to the original statement?
   (c) What is the contrapositive of the original statement?
   (d) Is (c) logically equivalent to the original statement?