5. (14 points) How many monomials of degree d can be formed using the variables $x_1, x_2, \ldots, x_n$? (For example, the monomials of degree 3 in the variables x_1, x_2, x_3, x_4 are:

$$x_1^3, \quad x_2^3, \quad x_3^3, \quad x_4^3, \quad x_1^2x_2, \quad x_1^2x_3, \quad x_1^2x_4, \quad x_2^2x_1, \quad x_2^2x_3, \quad x_2^2x_4,$$

$$x_3^2x_1, \quad x_3^2x_2, \quad x_3^2x_4, \quad x_4^2x_1, \quad x_4^2x_2, \quad x_4^2x_3, \quad x_1x_2x_3, \quad x_1x_3x_4, \quad x_1x_2x_4, \quad x_2x_3x_4.)$$

This problem is the same as how many types of bags of candy can be made if we have n flavors and each bag gets d pieces. (i.e. d picks $n-1$ switches)

$$\binom{d+n-1}{d}$$

6. (14 points) Prove that $\displaystyle\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.

We induct on n.
When $n=1$ the left side is 1. The right side is $\frac{1\cdot 2\cdot 3}{6} = 1$

Assume, by induction, that $\displaystyle\sum_{k=1}^{n-1} k^2 = \frac{(n-1)n(2n-1)}{6}$

We must show that $\displaystyle\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$. However,

$$\sum_{k=1}^{n} k^2 = \sum_{k=1}^{n-1} k^2 + n^2 \underset{\text{IH}}{=} \frac{(n-1)n(2n-1)}{6} + n^2 = \frac{n}{6}\left[(n-1)(2n-1) + 6n\right]$$

$$= \frac{n}{6}\left[2n^2 - 3n + 1 + 6n\right] = \frac{n}{6}\left[2n^2 + 3n + 1\right] = \frac{n}{6}\left[(n+1)(2n+1)\right]$$