20) Solve the recurrence relation

$a_{k+1} = a_k + k + 7 \quad a_0 = 0.$

Let $A = \sum_{k=0}^{\infty} a_k x^k$

$$\sum_{k=0}^{\infty} a_{k+1} x^k = \sum_{k=0}^{\infty} a_k x^k + \sum_{k=0}^{\infty} k x^k + 7\sum_{k=0}^{\infty} x^k$$

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$$

$$\frac{1}{(1-x)^2} = \sum_{k=0}^{\infty} k x^{k-1} \qquad \frac{x}{(1-x)^2} = \sum_{k=0}^{\infty} k x^k$$

$$\frac{1}{x} A = A + \frac{x}{(1-x)^2} + \frac{7}{1-x}$$

$$(1-x)A = \frac{x^2 + 7x(1-x)}{(1-x)^2}$$

$$A = \frac{-6x^2 + 7x}{(1-x)^3} = \frac{1}{(1-x)^3} + \frac{5}{(1-x)^2} + \frac{-6}{(1-x)}$$

$$\frac{+2}{(1-x)^3} = \sum_{k=0}^{\infty} k(k-1)x^{k-2} \qquad \frac{1}{(1-x)^3} = \frac{1}{2}\sum_{k=0}^{\infty} (k+2)(k+1)x^k$$

$$A = \sum_{k=0}^{\infty} \left[\frac{1}{2}(k^2+3k+2) + 5(k+1) - 6\right] x^k$$

$$\frac{k^2}{2} + \frac{13k}{2}$$

$$\therefore a_k = \frac{k^2}{2} + \frac{13k}{2}$$