) Let X_1, X_2, X_3, X_4 be variables. How many monomials of the form $X_1^{e_1} X_2^{e_2} X_3^{e_3} X_4^{e_4}$ have degree 12? (Each exponent e_i is a non-negative integer. The degree of the monomial is $e_1+e_2+e_3+e_4$.)

ans = # of words of length 15 made with 12 p's and 3 s's

$$= \binom{15}{3}$$

$\frac{15 \cdot 14 \cdot 13}{3 \cdot 2}$

(4) A code-word from the alphabet $\{0,1,2,3,4\}$ is legal if the number of 0's is odd. Find a recurrence relation which gives the number of legal words of length n.

Let a_n = # of legal words of length n

$a_2 = 8$

$a_1 = 1$

$a_n = 4 \cdot a_{n-1} + (5^{n-1} - a_{n-1})$

these start with 1, 2, 3 or 4

these start with 0

(15) Messages are words constructed from the alphabet $\{a,b,c\}$. It costs 1 dollar to send "a", 2 dollars to send "b" and 3 dollars to send "c". Find a recurrence relation which gives the number of messages which cost n dollars.

Let a_n = # of messages which cost n dollars.

$a_1 = 1$ ← word a

$a_2 = 2$ ← words aa, b

$a_3 = 4$ ← words: aaa, ab, ba, c

$a_n = a_{n-1} + a_{n-2} + a_{n-3}$

a 1st b 1st c 1st