

5) Solve the recurrence relation $a_n = 15a_{n-1} - 44a_{n-2}$, $a_0 = 0, a_1 = 7$.

$a_n = -4^n + 11^n$

$x^2 - 15x + 44 = 0$

$(x-11)(x-4) = 0$

$x = 11, 4$

$a_n = \lambda_1 4^n + \lambda_2 11^n$

$0 = \lambda_1 + \lambda_2$

$7 = 4\lambda_1 + 11\lambda_2$

$\therefore 7 = 4(-\lambda_2) + 11\lambda_2$

$7 = 7\lambda_2$

$1 = \lambda_2 \quad \lambda_1 = -1$

6) Solve the recurrence relation $a_n = a_{n-1} + a_{n-2} - a_{n-3}$, $a_0 = -9, a_1 = -8, a_2 = -3$.

$x^3 - x^2 - x + 1 = 0$

$(x-1)^2(x+1) = 0$

$x = 1, 1, -1$

$a_n = \lambda_1(1)^n + \lambda_2 n(1)^n + \lambda_3(-1)^n$

$-9 = \lambda_1 + \lambda_3$

$-8 = \lambda_1 + \lambda_2 - \lambda_3$

$-3 = \lambda_1 + 2\lambda_2 + \lambda_3$

$-3 - (-9) = 2\lambda_2$

$6 = 2\lambda_2$

$3 = \lambda_2$

$-9 = \lambda_1 + \lambda_3$

$-8 = \lambda_1 + 3 - \lambda_3$

$-17 = 2\lambda_1 + 3$

$\lambda_1 = -10$

$\lambda_3 = 1$

$a_n = -10 + 3n + (-1)^n$