

8 For each graph G , let $\omega(G)$ be the largest number p such that the complete graph on p letters is a subgraph of G .

(a) Give an inequality (of the form $\text{---} \leq \text{---}$) which relates $\omega(G)$ and $\chi(G)$.

(b) Give an example of a graph G such that your formula in (a) holds with $=$ in place of \leq .

(c) Give an example of a graph G such that your formula in (a) holds with $<$ in place of \leq .

(d) Prove that your formula in (a) holds for every graph G .

9 For each graph G , let $\Delta(G) = \max \{ \deg u \mid u \text{ is a vertex of } G \}$

(a) Give an inequality (of the form $\text{---} \leq \text{---}$) which relates $\Delta(G)$ and $\chi(G)$.

(b) Give an example of a graph G such that your formula in (a) holds with $=$ in place of \leq .

(c) Give an example of a graph G such that your formula in (a) holds with $<$ in place of \leq .

(d) Prove that your formula in (a) holds for every graph G .