5. (17 points) Find a recurrence relation for the number of ways to arrange identical Toyotas, identical Hondas, identical Fords, and identical Cadillacs into n spaces if Toyotas and Hondas each take up one space, Fords take up two spaces, and Cadillacs take up three spaces. Do not forget to give sufficient initial conditions.

Let a_n = the # of ways to arrange T, H, F, and C's as indicated into n spaces.

The collection of legal arrangements into n spaces may be partitioned as follows:
last car is T or last car is H or last car is F or last car is C

Thus, $a_n = a_{n-1} + a_{n-1} + a_{n-2} + a_{n-3}$

so $a_n = 2a_{n-1} + a_{n-2} + a_{n-3}$
$a_1 = 2$
$a_2 = 5$
$a_3 = 13$

Note The # of legal arrangements into n spaces with the last car an F is precisely a_{n-2}.

6. (17 points) What is the probability that a random arrangement of the letters in MISSISSIPPI has no consecutive vowels?

U is all arrangements of MIIIISSSSPP

$$\#(U) = \frac{11!}{4!\,4!\,2!}$$

S is all arrangements of MIIIISSSSPP with no consecutive vowels

$$\#(S) = \begin{pmatrix}\#\text{ of arrangements of}\\ 4\text{I's and 7 C's}\\ \text{with no consecutive I's}\end{pmatrix} \cdot \begin{pmatrix}\#\text{ of ways to}\\ \text{fill in 7 c's}\\ \text{with MSSSSPP}\end{pmatrix}$$

$$= \binom{8}{4}\frac{7!}{4!\,2!}$$

_C_C_C_C_C_C_C_C_
Pick 4 of the 8 _ to be I.
The other 4 disappear

$$\text{ans} = \frac{\binom{8}{4}\frac{7!}{4!\,2!}}{\frac{11!}{4!\,4!\,2!}}$$