3. (17 points) Write

$$\binom{100}{0} + \binom{101}{1} + \binom{102}{2} + \binom{103}{3} + \cdots + \binom{997}{897} + \binom{998}{898} + \binom{999}{899} + \binom{1000}{900}$$

as one binomial coefficient. Explain your work.

Ans = $\binom{1001}{900}$

Recall the empty wraper identity

$$\sum_{k=0}^{p} \binom{k+f-1}{k} = \binom{p+f}{p}$$

Take $p = 1000$ and $f = 101$.

4. (17 points) Write

$$\binom{500}{0}\binom{1000}{300} + \binom{500}{1}\binom{1000}{299} + \binom{500}{2}\binom{1000}{298} + \binom{500}{3}\binom{1000}{297} + \cdots$$
$$\cdots + \binom{500}{297}\binom{1000}{3} + \binom{500}{298}\binom{1000}{2} + \binom{500}{299}\binom{1000}{1} + \binom{500}{300}\binom{1000}{0}$$

as one binomial coefficient. Explain your work.

ANS = $\binom{1500}{300}$

Both numbers are the answer to the following problem:

A committee has 500 Men and 1000 Women. How many 300-member subcommittees can be formed?