

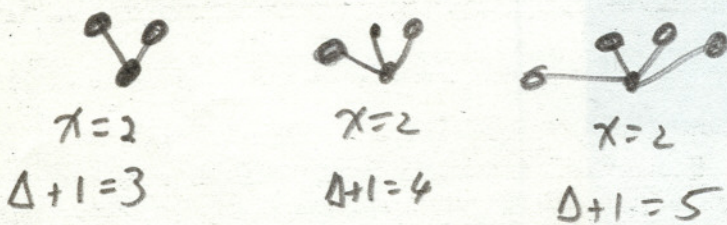
- (9) For each graph G , let $\Delta(G) = \max \{ \deg u \mid u \text{ is a vertex of } G \}$.
- (a) Give an inequality (of the form $_ \leq _$) which relates $\Delta(G)$ and $\chi(G)$.

$$\chi(G) \leq \Delta(G) + 1$$

- (b) Give an example of a graph G such that your formula in (a) holds with $=$ in place of \leq .

$$\begin{aligned} \chi &= 1 \\ \Delta &= 0 \end{aligned}$$

- (c) Give an example of a graph G such that your formula in (b) holds with $<$ in place of \leq .



- (d) Prove that your formula in (a) holds for every graph G .

Induct on the # of vertices of G .
 If G has one vertex, then G is \bullet and it works.

IH (suppose $\chi(G) \leq \Delta(G) + 1$ whenever # of vertices of $G < N$.)

Let G have N vertices. Let

G' be G with one vertex u removed.

IH says $\chi(G') \leq \Delta(G') + 1$

But $\Delta(G') \leq \Delta(G)$

so $\chi(G') \leq \Delta(G) + 1$

Color G' using $\Delta(G) + 1$ colors. u has at most $\Delta(G)$ nbds, in G so some color doesn't appear in his neighborhood. Color him that color. We have now colored G with $\leq \Delta(G) + 1$ colors.