8 For each graph G, let $\omega(G)$ be the largest number p such that the complete graph on p letters is a subgraph of G.

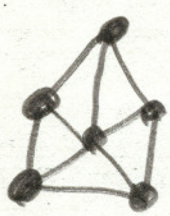
a Give an inequality (of the form $\underline{\omega} \leq \underline{\chi}$) which relates $\omega(G)$ and $\chi(G)$.

$$\omega(G) \leq \chi(G)$$

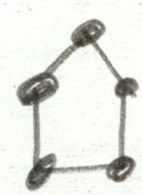
b Give an example of a graph G such that your formula in a holds with = in place of $\leq$.

In K_2  $\omega = 2$ $\chi = 2$

c Give an example of a graph G such that your formula in a holds with < in place of $\leq$.



$\omega = 3$
$\chi = 4$



$\omega = 2$
$\chi = 3$

d Prove that your formula in a holds for every graph G.

If each vertex from $x_1, \dots, x_p$ is connected to all of the others, then we must use p colors just to color $x_1, \dots, x_p$. We might need more colors elsewhere.