

The number  $a_1$  fits some place on the list  $b_1 \leq \dots \leq b_m$ . Let "r" be the house of the index with  $b_r \leq a_1 \leq b_{r+1}$ . We have made  $r+1$  comparisons so far. Complete this merge by merging  $\underbrace{a_2 \leq \dots \leq a_{n+1}}_{n \text{ #'s}}$  into  $\underbrace{b_{r+1} \leq \dots \leq b_m}_{m-r \text{ #'s}}$ .

$m-r$  #'s

The IH says that at most  $n + (m-r) + 1$  comparisons are used here. All together at most  $r+1 + n + (m-r) + 1 = n+m$  comparisons were made!

4. When  $n=1$ : Move the disk to rod C. 1 Move!  
Note:  $1 = 2^1 - 1$  ✓

When  $n=2$ : Move the small disk to rod B. Move the large disk to rod C. Move the small disk to rod C. 3 Moves! Note:  $3 = 2^2 - 1$  ✓

IH: Suppose that  $n$  disks can be moved from rod A to rod C (using rod B for storage) in  $2^n - 1$  moves!

Instructions for  $n+1$  rods: Move the smallest  $n$  disks from rod A to rod B (using rod C for storage). The IH says that this required at most  $2^n - 1$  moves.

Move the largest disk to rod C: 1 Move

Move the  $n$  smallest disk from rod B to rod C (using rod A for storage) IH says at most  $2^n - 1$  moves

But  $(2^n - 1) + 1 + (2^n - 1) = 2^{n+1} - 1$  ✓