The number $a_1$ fits some place on the list $b_1 \le \ldots \le b_m$. Let "r" be the name of the index with $b_r \le a_1 \le b_{r+1}$. We have made $r+1$ comparisons so far. Complete this merge by merging $\underbrace{a_2 \le \ldots \le a_{n+1}}_{n \text{ #'s}}$ into $\underbrace{b_{r+1} \le \ldots \le b_m}_{m-r \text{ #'s}}$.

The IH says that at most $n+(m-r)+1$ comparisons are used here. All together at most

$$r+1 \quad + \quad n+(m-r)+1 \quad = \quad n+m$$

comparisons were made!

4. When $n=1$: Move the disk to rod C. 1 Move!

Note: $1 = 2^1 - 1$ ✓

When $n=2$; Move the small disk to rod B. Move the large disk to rod C. Move the small disk to rod C. 3 Moves! Note: $3 = 2^2 - 1$ ✓

IH: Suppose that $n$ disks can be moved from rod A to rod C (using rod B for storage) in $2^n - 1$ moves!

<u>Instructions for $n+1$ rods</u>: Move the smallest $n$ disks from rod A to rod B (using rod C for storage). The IH says that this required at most $2^n - 1$ moves.

Move the largest disk to rod C: 1 Move

Move the $n$ smallest disk from rod B to rod C (using rod A for storage) IH says at most $2^{n-1}$ moves

But $(2^n - 1) + 1 + (2^n - 1) = 2^{n+1} - 1$ ✓