Math 574 1992 Exam 1 Solutions.

(1) <u>$n=1$</u> I must prove
$$1 = \frac{1(1+1)(2(1)+1)}{6}.$$
The right side is $\frac{2\cdot 3}{6} = 1.$ ✓

By induction I assume that $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$. (*)

I must show that $\sum_{k=1}^{n+1} k^2 = \frac{(n+1)(n+2)(2n+3)}{6}$ (**)

The Left side of (**) $= \sum_{k=1}^{n} k^2 + (n+1)^2 \underset{\text{by (*)}}{=} \frac{n(n+1)(2n+1)}{6} + (n+1)^2$

$$= (n+1)\left[\frac{2n^2+n}{6} + \frac{6n+6}{6}\right] = \frac{(n+1)(2n^2+7n+6)}{6} = \frac{(n+1)(n+2)(2n+3)}{6}. ✓$$

(2) <u>$n=1$</u>: I must prove $2 = (1+\sqrt{5})^1 + (1-\sqrt{5})^1$. This is obvious.

<u>$n=2$</u>: I must prove that $12 = (1+\sqrt{5})^2 + (1-\sqrt{5})^2$.

The right side is $1+\cancel{2\sqrt{5}}+5 + 1-\cancel{2\sqrt{5}}+5 = 12$ ✓

Let n be a fixed integer with $3 \le n$. By induction we assume that
$$a_k = (1+\sqrt{5})^k + (1-\sqrt{5})^k$$

For all k with $1 \le k \le n-1$.
We must prove that
$$a_n = (1+\sqrt{5})^n + (1-\sqrt{5})^n;$$