

**Math 574, 1992, Final Exam**

PRINT Your Name: \_\_\_\_\_

There are 11 problems on 6 pages. The exam is worth a total of 150 points.

SHOW your work. *CIRCLE* your answers. CHECK your answers.

1. (13 points) How many integer solutions are there to the equality

$$x_1 + x_2 + x_3 + x_4 = 14,$$

with  $1 \leq x_1$ ,  $2 \leq x_2$ ,  $0 \leq x_3$ , and  $0 \leq x_4$ ?

2. (13 points) A class contains 50 women and 20 men. Ten people are chosen at random. What is the probability that none are men?
3. (13 points) A True – False test consists of ten questions. If a student selects answers at random, then what is the probability that the student will guess at least 7 correct answers?
4. (13 points) Eight friends decide to have their picture taken. How many ways are there to arrange all eight people in a straight line, if John refuses to stand next to Mary?
5. (14 points) How many monomials of degree  $d$  can be formed using the variables  $x_1, x_2, \dots, x_n$ ? (For example, the monomials of degree 3 in the variables  $x_1, x_2, x_3, x_4$  are:

$$\begin{aligned} &x_1^3, \quad x_2^3, \quad x_3^3, \quad x_4^3, \quad x_1^2x_2, \quad x_1^2x_3, \quad x_1^2x_4, \quad x_2^2x_1, \quad x_2^2x_3, \quad x_2^2x_4, \\ &x_3^2x_1, \quad x_3^2x_2, \quad x_3^2x_4, \quad x_4^2x_1, \quad x_4^2x_2, \quad x_4^2x_3, \quad x_1x_2x_3, \quad x_1x_3x_4, \quad x_1x_2x_4, \quad x_2x_3x_4.) \end{aligned}$$

6. (14 points) Prove that  $\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$ .
7. (14 points) Prove that the sum of the fifth powers of any five consecutive nonnegative integers is divisible by 25.
8. (14 points) Write  $\binom{300}{300} + \binom{301}{300} + \binom{302}{300} + \binom{303}{300} + \dots + \binom{2997}{300} + \binom{2998}{300} + \binom{2999}{300} + \binom{3000}{300}$  as one binomial coefficient. Explain your work.
9. (14 points) Write  $\binom{600}{0} \binom{1100}{700} + \binom{600}{1} \binom{1100}{699} + \binom{600}{2} \binom{1100}{698} + \binom{600}{3} \binom{1100}{697} + \dots$   
 $\dots + \binom{600}{597} \binom{1100}{103} + \binom{600}{598} \binom{1100}{102} + \binom{600}{599} \binom{1100}{101} + \binom{600}{600} \binom{1100}{100}$  as one binomial coefficient. Explain your work.
10. (14 points) Solve the recurrence relation  $a_n - 5a_{n-1} + 8a_{n-2} - 4a_{n-3} = 3$ , with  $a_0 = 0$ ,  $a_1 = 3$ , and  $a_2 = 13$ .

11. (14 points) Find a CLOSED formula for  $\sum_{k=0}^n k^4$ .