Math 554, Exam 2, Summer 2004

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.

There are 10 problems. Each problem is worth 5 points. The exam is worth a total of 50 points.

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

I will leave your exam outside my office door by noon tomorrow, you may pick it up any time between then and the next class.

I will post the solutions on my website shortly after the class is finished.

1. Define *supremum*. Use complete sentences.

2. Define *limit point*. Use complete sentences.

3. State the least upper bound axiom.

4. State either version of the Bolzano-Weierstrass Theorem.

5. Let $f\colon X \to Y$ and $g\colon Y \to Z$ be functions with f one-to-one and g one-to-one, prove that the function $g \circ f\colon X \to Z$ is one-to-one.

6. Give an example of a bounded set with exactly three limit points.

7. For each natural number n, let I_n be the open interval $(0, \frac{1}{n})$. What is $\bigcap_{n=1}^{\infty} I_n$? Prove your answer.

8. Let $\{a_n\}$ be a sequence which converges to a and $\{b_n\}$ be a sequence which converges to b. Prove that the sequence $\{a_n b_n\}$ converges to ab.

9. Let $\{b_n\}$ be a sequence which converges to b, with $b \neq 0$. Prove that the sequence $\{\frac{1}{b_n}\}$ converges to $\frac{1}{b}$.

10. Let $\{p_n\}$ be a bounded sequence of real numbers and let $p \in \mathbb{R}$ be such that every convergent subsequence of $\{p_n\}$ converges to p. Prove that the sequence $\{p_n\}$ converges to p.