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Quiz for June 14, 2006

a. Let $\{\mathcal{O}_\alpha \mid \alpha \in A\}$ be a set of open subsets of $\mathbb{R}$. Prove that $\bigcup_{\alpha \in A} \mathcal{O}_\alpha$ is an open subset of $\mathbb{R}$.

b. Give an example of a set of open subsets $\{\mathcal{O}_\alpha \mid \alpha \in A\}$ of $\mathbb{R}$ with $\bigcap_{\alpha \in A} \mathcal{O}_\alpha$ not an open subset of $\mathbb{R}$.

ANSWER:

a. Let p be an element of $\bigcup_{\alpha \in A} \mathcal{O}_\alpha$. Thus, p is in $\mathcal{O}_{\alpha_0}$ for some $\alpha_0 \in A$. The set $\mathcal{O}_{\alpha_0}$ is open, so there exists $\varepsilon > 0$ with $N_\varepsilon(p) \subseteq \mathcal{O}_{\alpha_0}$. Thus, $N_\varepsilon(p) \subseteq \bigcup_{\alpha \in A} \mathcal{O}_\alpha$.

b. For each natural number n, let $I_n = (0, 1 + \frac{1}{n})$. It is clear that each open interval I_n is an open set in $\mathbb{R}$. It is also clear that $\bigcap_{n \in \mathbb{N}} I_n = (0, 1]$; which is not an open subset of $\mathbb{R}$. Indeed, $N_\varepsilon(1)$ is not contained in $(0, 1]$ for any $\varepsilon > 0$.