

## Notes on The Final Exam, Math 554, Summer 2004

1. The final exam is Wednesday, June 30 in our usual room starting at 8:00 AM. The exam is comprehensive, worth 100 points. All theorems and definitions that you have been responsible for on past exams remain fair game. I strongly encourage you to DO old problems (from homework and exams) on blank paper without notes or books. (AFTER you feel happy with your end of June argument, then compare it to an argument that is generally accepted. If you are not certain if your latest argument is any good, feel free to give it to me. I can check it pretty quickly.) Feel free, of course, to attack other related problems from the textbook (or other similar textbooks) also. Also, if you come upon something that used to make sense, but doesn't make sense anymore, or something that never made sense (but you memorized it anyhow), please ask about it.
2. The final exam covers sections: 1.4, 1.5, 1.7, 2.1–2.4, 2.6, 3.1, 3.2, 4.1, 4.2, and 5.1.
3. Be able to define “bounded above”, “upper bound”, “supremum”, “one-to-one”, “onto”, “have the same cardinality”, “finite”, “countable”, “uncountable”, “sequence”, “the sequence converges”, “the limit of a sequence”, “monotone sequence”, “limit point”, “Cauchy sequence”, “open set”, “closed set”, “closure”, “dense set”, “compact set”, “limit of a function”, “continuous”, “derivative”.
4. Be able to state: the least upper bound property of the real numbers, the Archimedean Property of the real numbers, the theorem about monotone sequences, the nested interval property, a version of the Bolzano-Weierstrass Theorem, the Heine-Borel Theorem, and the characterization of closed sets in terms of limit points.
5. Be able to prove:  $(0, 1)$  is uncountable, the set of positive rational numbers is countable, the limit of a product of two sequences is the product of the limits of the individual sequences, the theorem about monotone sequences, the nested interval property, a version of the Bolzano-Weierstrass Theorem, the Heine-Borel Theorem, the characterization of closed sets in terms of limit points, that Cauchy sequences are bounded, that Cauchy sequences converge, that compact sets are bounded, that compact sets are closed, that the continuous image of a compact set is compact, the intermediate value theorem, that a continuous function defined on a closed interval has a maximum and a minimum, the product rule for derivatives, the fact that a differentiable function is continuous.
6. You are able to all of the problems on all of the old exams.