$N_{x_1}, \dots, N_{x_m}$ covers K. It follows that

$f(N_{x_1}), \dots, f(N_{x_m})$ covers $f(K)$.

But N_{x_i} was constructed so that $f(N_{x_i}) \subseteq U_{x_i} \in \mathcal{U}$

Thus $U_{x_1}, \dots, U_{x_m}$ is an open cover of $f(K)$,
and $f(K)$ is compact

(12) ~~Yes~~ If E_a is open for all a, then $\bigcup E_a$ is an open set.
Given $x \in \bigcup E_a$, we have $x \in E_{a_0}$ for some a_0, so $\exists\, \varepsilon$ with $N_\varepsilon(q) \subseteq E_{a_0}$.
Thus $N_\varepsilon(q) \subseteq \bigcup E_a$. Thus $\bigcup E_a$ is an open set

(13) For each $n \in \mathbb{N}$ let $E_n = (-\frac{1}{n}, 1)$. We see each E_n is an open set but $\bigcap_{n=1}^{\infty} E_n = [0,1)$ which is not an open set.

NO.

(14) (a) Suppose a_1 and a_2 are in A with $f(a_1) = f(a_2)$ in B.
Apply g to both sides to get $g(f(a_1)) = g(f(a_2))$
But this tells me $a_1 = a_2$ because $g(f(a)) = a \;\forall a \in A$.
Thus f is 1-1

(b) Given $a \in A$. I am told that $g(f(a)) = a$
Thus $f(a) \in B$ and $g(f(a)) = a$, Thus g is onto

(c) Consider $f : \{1\} \longrightarrow \{2,3\}$ defined by $f(1) = 2$
and $g : \{2,3\} \longrightarrow \{1\}$ defined by $g(2) = g(3) = 1$
we see that $(g \circ f)(1) = g(2) = 1$.
We see that g is not one-to-one, because $g(2) = g(3)$
but $2 \neq 3$.