109 $\lim_{x \to 2} f(x) = 8$

Given $\varepsilon > 0$.

$$|f(x)-8| = \begin{cases} |2x^2 - 8| = 2|x+2|\,|x-2| & \text{if } x \text{ is rational} \\ |5x-2-8| = 5|x-2| & \text{if } x \text{ is irrational} \end{cases}$$

I will only think about $\delta < 1$ so $|x-2| < \delta \Rightarrow |x| < 3$

so $|x-2| < \delta \Rightarrow |x+2| \le |x| + 2 = 5$

Thus

$$|f(x)-8| \le \begin{cases} 10|x-2| & \text{if } x \text{ is rational} \\ 5|x-2| & \text{if } x \text{ is irrational} \end{cases}$$

Thus $|f(x)-8| \le 10|x-2|$

Take $\delta =$ the minimum of $\{\frac{\varepsilon}{10}, 1\}$

$|x-2| < \delta \Rightarrow |f(x)-8| \le 10|x-2| < 10\delta \le 10 \cdot \frac{\varepsilon}{10} = \varepsilon$

It follows that $\lim_{x \to 2} f(x) = 8$

106 Consider a sequence of rational numbers $a_1, a_2, \ldots$ which converges to 1. $\lim_{n \to \infty} f(a_n) = 2$. Consider a sequence of irrational numbers $b_1, b_2, \ldots$ which converges to 1. $\lim_{n \to \infty} f(b_n) = 3$

$2 \ne 3$ so $\lim_{x \to 1} f(x)$ Does not exist.

(11) Let K be a compact set and $f: K \to \mathbb{R}$ be a continuous function. Fix an open cover $\mathcal{U}$ of $f(K)$. Let $x \in K$, then $f(x) \in f(K)$ so $f(x) \in U_x$ for some $U_x \in \mathcal{U}$. The function f is continuous so $\exists$ an open neighborhood N_x of x with $f(N_x) \subseteq U_x$. $\{N_x \mid x \in K\}$ is an open cover of the compact set K so $\exists\, x_1, \ldots, x_m$ so that