Math 554 Summer 2000 Final Exam

① The subset E of $\mathbb{R}$ is an open set if for all $x \in E$ there exists $\varepsilon > 0$ such that $N_\varepsilon(x) \subseteq E$.

② The subset E of $\mathbb{R}$ is a compact set if every open cover of E contains a finite subcover.

③ Let $f: E \to \mathbb{R}$ be a function which is defined on a subset E of $\mathbb{R}$. Let p be a limit point of E and L be an element of E. Then $\lim_{x \to p} f(x) = L$ means for all $\varepsilon > 0$, there exists $\delta > 0$ such that if $x \in E$ with $0 < |x-p| < \delta$, then $|f(x) - L| < \varepsilon$.

④ The "limit of the sequence $\{a_n\}_{n=1}^{\infty}$ is equal to L" means that for all $\varepsilon > 0$, there exists an integer n_0 such that $|a_n - L| < \varepsilon$ for all $n \geq n_0$

⑤ Let E be a non-empty subset of $\mathbb{R}$. If E is bounded from above then the supremum of E is a number a such that a is an upper bound of E and no smaller number is an upper bound of E.

⑥ Let E be a set of real numbers. The number p is called a limit point of E if for all $\varepsilon > 0$, $E \cap N_\varepsilon(p)$ contains an element x with $x \neq p$.

⑦ The Heine-Borel Theorem states that the closed interval $[a,b]$ is compact whenever $a < b$.

⑧ The Bolzano-Weierstrass Theorem states that every bounded sequence contains a convergent subsequence.

⑨ Least Upper Bound Property Every non-empty subset of real numbers which is bounded above has a supremum.