

(7) We will prove that the sequence  $\{a_n b_n\}$  diverges to  $+\infty$ . Fix  $M, a$  <sup>positive</sup> real numbers. We will show that  $\exists n_0$  s.t.  $n \geq n_0 \Rightarrow M < a_n b_n$ .

The sequence  $\{b_n\}$  converges to  $\frac{1}{10^6}$ , so  $\exists n_1$  such that

$$\text{for all } n \geq n_1 \quad \frac{1}{10^7} \leq b_n.$$

The sequence  $\{a_n\}$  diverges to  $+\infty$ , so  $\exists n_2$  s.t.

$$\text{for all } n \geq n_2 \quad M 10^7 \leq a_n$$

Let  $n_0$  be the maximum of  $n_1$  and  $n_2$ . If  $n \geq n_0$

then  $\begin{cases} \frac{1}{10^7} \leq b_n \\ M 10^7 \leq a_n. \end{cases}$  It follows that  $M 10^7 \cdot \frac{1}{10^7} \leq a_n b_n$ ,

as desired. Thus the sequence  $\{a_n b_n\}$  diverges to  $+\infty$ .