Consequently, if $n \geq \max\{n_0, n_1\}$ then

$$|(a+b) - (a_n + b_n)| \leq |a - a_n| + |b - b_n| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} < \varepsilon.$$

We conclude that the sequence $\{a_n + b_n\}$ converges to $a + b$.

(8a) Given $\varepsilon > 0$, we see that

$$|a^2 - a_n^2| = |a^2 - aa_n + aa_n - a_n^2| \leq |a^2 - aa_n| + |aa_n - a_n^2|$$

$$\leq |a|\,|a - a_n| + |a - a_n|\,|a_n| \leq |a - a_n|\,(|a| + 100)$$

The sequence $\{a_n\}$ converges to a so $\exists\, n_0$ s.t. $|a - a_n| < \frac{\varepsilon}{|a| + 100}$ for all $n \geq n_0$. We conclude that

$$|a^2 - a_n^2| \leq |a - a_n|\,(|a| + 100) < \frac{\varepsilon}{(|a| + 100)}(|a| + 100) < \varepsilon,$$

for $n > n_0$.

(b) Let $a_n = (-1)^n$ for all n, so $a_n = 1$ when n is even and $a_n = (-1)$ when n is odd. This sequence diverges. The sequence $\{a_n^2\}$ is $1, 1, 1, 1, \ldots$) which converges to 1.