(6) Let $E = \{x \in \mathbb{R} \mid 0 \leq x < 1\}$

$\inf E = 0 \in E$

$\sup E = 1 \notin E$

(7) Let $\alpha = \inf A$ and $\beta = \inf B$

Step 1 $\alpha\beta$ is a lower bound for $A \cdot B$

Take an arbitrary element of $A \cdot B$. It has the form
ab for some $a \in A$ and some $b \in B$.

$\alpha = \inf A \implies \alpha \leq a$

$\beta = \inf B \implies \beta \leq b$

all of the numbers $\alpha, \beta, a,$ and b are positive hence

$$\alpha\beta \leq a\beta \leq ab$$

we have shown that $\alpha\beta$ is a lower bound for $A \cdot B$

Step 2 We must show that if γ is any number with
$\alpha\beta < \gamma$ then there exists an element of $A \cdot B$ which
is smaller than γ.

I will find an element of A which is "very close" to α and
an element of B which is "very close" to β. The product
of these two will be very close to $\alpha\beta$ and therefore
smaller than γ.

Lets. nail down what we really mean by "very close".
Let $\varepsilon_1 > 0$ be small (We'll decide how small later) and
$\varepsilon_2 > 0$ also be small

Since $\alpha < \alpha + \varepsilon_1$ we know that $\alpha + \varepsilon_1$ is not a lower bound for A
so there exists $a \in A$ with $\alpha + \varepsilon_1 > a$

Also $\beta + \varepsilon_2$ is not a lower bound for B so there exists $b \in B$
with $b < \beta + \varepsilon_2$