Math 554 Summer 2000 Exam 1

(1)

① The real number L is a lower bound of the non-empty set of real numbers E if $L \le x$ for all $x \in E$.

② The real number L is the infimum of the non-empty set of real numbers E if L is a lower bound for E and whenever $\beta > L$, then β is not a lower bound for E.

③ Every non-empty set of real numbers which is bounded above has a supremum.

④ If x and y are real numbers with $x > 0$, then there exists a positive integer n with $y < nx$.

⑤ Let $\alpha = \sup L$ and $\beta = \inf E$.

Step 1 $\beta = \inf E$ hence β is a lower bound for E and therefore $\beta \in L$. On the other hand, $\alpha = \sup L$ so α is an upper bound for L. It follows that $\beta \le \alpha$.

Step 2 Suppose $\beta < \alpha$. (We expect to reach a contradiction.)

$\beta < \alpha = \sup L$ so β is not an upper bound for L so $\exists\, l \in L$ with $\beta < l$.

On the other hand, $\beta = \inf E$ and $\beta < l$ so l can not be a lower bound for E.

We have $l \in L$ with l not a lower bound for E but this is a contradiction because every element of L is a lower bound for E. This is a contradiction, so our supposition ~~is false~~ $\beta < \alpha$ is false.

We have $\beta \le \alpha$ but β is not $< \alpha$. The only remaining possibility is $\beta = \alpha$.