Math 544 Final Exam Summer 2000

Use the paper provided. Problem 10 is worth 12 points. Problem 14 is worth 10 points. The other problems are worth 6 points each.

If you e-mail me requesting that I send you your grade, I will do it. Or, get your grade from VIP or TIPS.

1. Define "open set".

2. Define "compact set".

3. Let $f\colon E \to \mathbb{R}$ be a function which is defined on a subset E of $\mathbb{R}$. Define $\lim\limits_{x\to p} f(x) = L$. (Be sure to tell me what kind of a thing p is, and what kind of a thing L is.)

4. Let $a_1, a_2, \ldots$ be a sequence of real numbers, and let L be a real number. Define the expression, "the limit of the sequence $\{a_n\}_{n=1}^{\infty}$ is equal to L".

5. Define "supremum".

6. Define "limit point".

7. State the Heine-Borel Theorem.

8. State either version of the Bolzano-Weierstrass Theorem.

9. State the least upper bound property of the real numbers.

10. Let $f(x) = \begin{cases} 2x^2 & \text{if } x \text{ is rational} \\ 5x-2 & \text{if } x \text{ is irrational.} \end{cases}$
    (a) Does $\lim\limits_{x\to 2} f(x)$ exist? If the limit exists, find the limit and prove your answer. If the limit does not exist, give a coherent proof why it doesn't exist.
    (a) Does $\lim\limits_{x\to 1} f(x)$ exist? If the limit exists, find the limit and prove your answer. If the limit does not exist, give a coherent proof why it doesn't exist.

11. Prove that the continuous image of a compact set is compact.

12. Let A be a set. For each $a \in A$, let E_a be an open subset of $\mathbb{R}$. Is the union $\bigcup\limits_{a\in A} E_a$ always an open set? If your answer is yes, prove it. If your answer is no, give an example.

13. Let A be a set. For each $a \in A$, let E_a be an open subset of $\mathbb{R}$. Is the intersection $\bigcap\limits_{a\in A} E_a$ always an open set? If your answer is yes, prove it. If your answer is no, give an example.

14. Let A and B be non-empty sets, and let $f\colon A \to B$ and $g\colon B \to A$ be functions. Suppose that $g \circ f$ is the identity function on A. (In other words, $g(f(a)) = a$ for all a in A.)
    (a) Prove that f is one-to-one.
    (b) Prove that g is onto.
    (c) Give an example to show that g does not have to be one-to-one. (Your example can be very small.)

15. Let f and g be functions from $\mathbb{R}$ to $\mathbb{R}$, and let p, A, and B be real numbers. Suppose that $\lim_{x \to p} f(x) = A$ and $\lim_{x \to p} g(x) = B$. Prove $\lim_{x \to p} f(x)g(x) = AB$.