

Math 550, Final Exam, Spring 2013

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: turn the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 100 points. Write coherent correct explanations. Write in complete sentences. CIRCLE your answer, when this makes sense. **No Calculators or Cell phones.**

- (12 points) Compute $\int_0^1 \int_x^1 e^{y^2} dy dx$. Explain thoroughly.
- (11 points) Compute $\frac{d}{dx} \int_a^x \int_c^d f(x, y, z) dz dy$. Explain thoroughly.
- (11 points) Find the volume of the region bounded by $z = x^2 + y^2$ and $z = 10 - x^2 - 2y^2$. Explain thoroughly.
- (11 points) Let $D^* = [0, 1] \times [0, 1]$ and define T on D^* by $T(u, v) = (-u^2 + 4u, v)$. Find the image of T . Explain thoroughly.
- (11 points) Let D be the parallelogram with vertices $(1, 2)$, $(2, 3)$, $(4, 6)$, and $(3, 5)$. Compute $\int \int_D x dx dy$. Explain thoroughly.
- (11 points) Compute the path integral of $f(x, y, z) = xyz$ over the path $c(t) = (\cos t, \sin t, t)$ for $0 \leq t \leq \frac{\pi}{2}$. Explain thoroughly.
- (11 points) Consider the force field $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Compute the work done in moving a particle along the parabola $y = x^2$, $z = 0$, from $x = -1$ to $x = 2$. Explain thoroughly.
- (11 points) Find the area of the part of the unit sphere that satisfies $z \geq \sqrt{x^2 + y^2}$. Explain thoroughly.
- (11 points) Let the velocity field of a fluid be described by $\mathbf{F} = \mathbf{i} + x\mathbf{j} + z\mathbf{k}$ (measured in meters per second). Compute how many cubic meters of fluid per second are crossing the surface described by $x^2 + y^2 + z^2 = 1$ with $z \geq 0$. Explain thoroughly.