PRINT Your Name:___________________________________

Quiz for April 18, 2005

1. Let $K \subseteq \mathbb{C}$ be fields, and let $f(x)$ be an irreducible polynomial in $K[x]$. Prove that $f(x)$ has DISTINCT roots in $\mathbb{C}$.

ANSWER: This is a proof by contradiction. Suppose $\alpha \in \mathbb{C}$ is a root of $f(x)$ of multiplicity at least 2. Let I be the ideal in $K[x]$ of all polynomials $g(x)$ with $g(\alpha) = 0$. We see that $f \in I$. On the other hand, f generates a maximal ideal of $K[x]$; so $I = (f)$. We notice that the derivative $f'(x)$ is a polynomial in $K[x]$ with less degree than $f(x)$. On the other hand, if $f(x) = (x-\alpha)^2 g(x)$ in $\mathbb{C}[x]$, then the product rule tells us that $f'(x) = (x-\alpha)^2 g'(x) + 2(x-\alpha)g(x)$; hence, $f'(\alpha) = 0$, and $f' \in I = (f)$. This is impossible because f' is not identically zero, so f' can not be a multiple of f in $K[x]$.