Math 547, Exam 4, Spring , 2005

The exam is worth 50 points. Each problem is worth 10 points.

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Take enough space for each problem. Turn in your solutions in the order: problem 1, problem 2, ... ; although, by using enough paper, you can do the problems in any order that suits you.

I will e-mail your grade to you as soon as I finish grading the exams.

If you want me to leave your exam outside my door (so that you can pick it up before Wednesday's class), then **TELL ME** and I will do it. The exam will be there as soon as I e-mail your grade to you.

I will post the solutions on my website later today.

1. Let $K \subseteq L$ be fields, $f(x)$ be a polynomial in $K[x]$, $\sigma \in \operatorname{Aut}_K L$, and $\ell \in L$. Suppose that $f(\ell) = 0$. Prove $f(\sigma(\ell)) = 0$. Give all details.

2. Let $K \subseteq L$ be fields, $f(x)$ be an irreducible polynomial of $K[x]$, and α_1 and α_2 be elements of L with $f(\alpha_1) = f(\alpha_2) = 0$. Prove that there exists a ring isomorphism $\sigma\colon K[\alpha_1] \to K[\alpha_2]$ with $\sigma(\alpha_1) = \alpha_2$ and $\sigma(k) = k$ for all $k \in K$. Give all details.

3. State the Fundamental Theorem of Galois Theory. Please give hypotheses and conclusions.

4. Let F be the splitting field of $f(x) = x^3 - 2$ over $\mathbb{Q}$. Find all fields K with $\mathbb{Q} \subseteq K \subseteq F$. Give complete details.

5. We know that $x^9 - 1 = (x^3 - 1)(x^6 + x^3 + 1)$. We also know that $g(x) = x^6 + x^3 + 1$ is irreducible over $\mathbb{Q}$. (There is no need to re-prove these facts.) Let F be the splitting field of $g(x)$ over $\mathbb{Q}$. Find $\operatorname{Aut}_{\mathbb{Q}} F$. Be sure to tell me the elements of $\operatorname{Aut}_{\mathbb{Q}} F$ as well as the group structure. Give complete details.