Math 547, Exam 3, Spring , 2005

The exam is worth 50 points.

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Take enough space for each problem. Turn in your solutions in the order: problem 1, problem 2, ... ; although, by using enough paper, you can do the problems in any order that suits you.

I will e-mail your grade to you as soon as I finish grading the exams.

If you want me to leave your exam outside my door (so that you can pick it up before Wednesday's class), then **TELL ME** and I will do it. The exam will be there as soon as I e-mail your grade to you.

I will post the solutions on my website later today.

1. (6 points) Define *maximal ideal*.

2. (6 points) Define *Principal Ideal Domain*.

3. (6 points) Define *irreducible element*.

4. (8 points) Prove that $\mathbb{Q}[x]$ is a Principal Ideal Domain.

5. (8 points) Let $\alpha = e^{\frac{2\pi i}{9}}$ and let $\phi\colon \mathbb{Q}[x] \to \mathbb{C}$ be the function which is given by $\phi(f(x)) = f(\alpha)$. All of us know that this function is a ring homomorphism; you do not have to show me a proof. What is the kernel of ϕ? Prove your answer.

6. (8 points) Let M be a maximal ideal of the ring R. Prove that $\frac{R}{M}$ is a field.

7. (8 points) Suppose $K \subseteq F \subseteq E$ are fields with α_1, α_2 a basis of F over K and β_1, β_2 a basis of E over F.
   (a) What is a basis for E over K?
   (b) Prove your answer to (a).