Math 547, Exam 2, Spring , 2005

The exam is worth 50 points.

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Take enough space for each problem. Turn in your solutions in the order: problem 1, problem 2, ... ; although, by using enough paper, you can do the problems in any order that suits you.

I will e-mail your grade to you as soon as I finish grading the exams.

If you want me to leave your exam outside my door (so that you can pick it up before Monday's class), then **TELL ME** and I will do it. The exam will be there as soon as I e-mail your grade to you.

I will post the solutions on my website later today.

1. (8 points) Give an example of a ring R and an ideal I with I not a principal ideal. Explain.

2. (8 points) Prove that $\mathbb{Q}[x]$ is a Principal Ideal Domain.

3. (8 points) Is $\frac{\mathbb{Q}[x]}{(x^2-1)}$ a domain? Explain.

4. (8 points) Let $\alpha = e^{\frac{2\pi i}{6}}$ and let $\phi\colon \mathbb{Q}[x] \to \mathbb{C}$ be the function which is given by $\phi(f(x)) = f(\alpha)$. All of us know that this function is a ring homomorphism; you do not have to show me a proof. What is the kernel of ϕ? Prove your answer.

5. (9 points) Let $\phi\colon R \to S$ be a ring homomorphism. Suppose that $\ker\phi = \{0\}$. Prove that ϕ is one-to-one.

6. (9 points) Let M be an ideal of the ring R. Suppose that $M \neq R$ and that R is the only ideal of R which properly contains M. Prove that $\frac{R}{M}$ is a field.