MATH 547
ALGEBRAIC STRUCTURES II

Math 547 is the continuation of Math 546 and will use the same textbook, "Abstract Algebra", Second edition, by Beachy and Blair. Math 546 is a course about groups. Math 547 is about rings and fields. A field is a set F with two operations, usually called addition and multiplication. Under addition, F is an abelian group, with an identity element called 0. Under multiplication, $F \setminus \{0\}$, is an abelian group. The distributive axiom describes the interplay between the two operations. A ring is a set with two operations. Some of the field axioms hold in a ring. Some examples of fields are: the set of rational numbers, the set of real numbers, and the set of complex numbers. Every field is automatically a ring. The set of integers is a good example of a ring which is not a field. If R is a ring, then the set of all polynomials $\{f(x)\}$ with coefficients from R is another ring.

In Math 547, once we have finished the preliminary material, we will focus our attention on algebraic number fields. For each polynomial $f(x)$, with integer coefficients, we will study the smallest subfield of complex numbers which contains all of the roots of $f(x) = 0$. This field is called the splitting field of $f(x)$. Each such polynomial $f(x)$ corresponds to a finite group of permutations, called the Galois group of $f(x)$. The Fundamental Theorem of Galois Theory exhibits a one-to-one correspondence between the subgroups of the Galois group of $f(x)$ and the subfields of the splitting field of $f(x)$. Some of the highlights of our study will be:

1. Classical ruler and compass constructions. We will prove that it is impossible to solve three of the classical problems of ruler and compass construction. We will give a complete proof that there does not exist a ruler and compass construction for trisecting an angle. We will give a complete proof that there does not exist ruler and compass construction for doubling the cube. We will also show why there does not exist a ruler and compass construction for squaring the circle. This proof, however, will remain incomplete. One must prove elsewhere that π is a transcendental number.

2. The Fundamental Theorem of Algebra. We will give a complete algebraic proof of the well known fact from High School Algebra that every polynomial with integer coefficients has a complex root.

3. Polynomials of degree three and four. We will learn cubic and quartic analogues of the quadratic formula.

4. Polynomials of degree five. Our ultimate goal is to learn Galois' theorem that there does not exist a formula for expressing the roots of a general fifth degree polynomial equation as roots of roots of roots, etc., of algebraic expressions which involve the coefficients of the equations.