Math 546 Summer 2002 Final Exam

PRINT Your Name:________________________________

There are 20 problems on 10 pages. Each problem is worth 5 points.

I will not grade this exam until Tuesday. **Get your course grade from VIP.** The grade will be available from VIP as soon as I finish grading the exams.

I will post an answer key on my web site: www.math.sc.edu; click on faculty directory; click on kustin; click on teaching; click on math 546. The key will be posted shortly after the exam is completed.

1. Define "group isomorphism". Use complete sentences.

2. Define "normal subgroup". Use complete sentences.

3. Define "centralizer". Use complete sentences.

4. Define "center". Use complete sentences.

5. Define "cyclic group". Use complete sentences.

6. State and PROVE Lagrange's Theorem.

7. PROVE that every subgroup of $(\mathbb{Z}, +)$ is cyclic. I do NOT want you to prove a more general statement. I want you to prove the statement I have written. I want you to use notation which is appropriate to the **additive** group $\mathbb{Z}$.

8. Write down four groups. Each group is to have eight elements. None of the groups is to be isomorphic to any of the others. Explain thoroughly.

9. Let $\mathbb{R}^{\text{pos}}$ represent the group of positive real numbers under multiplication. Prove that the groups $(\mathbb{R}, +)$ and $(\mathbb{R}^{\text{pos}}, \times)$ are isomorphic.

10. Give an example of a subgroup of S_4 which has six elements. Explain.

11. Give an example of a subgroup of $(\mathbb{C} \setminus \{0\}, \times)$ which has six elements. Explain.

12. How many elements of S_5 have order 2? Explain.

13. Express the permutation $(6,9)(1,2)(4,9,7)(4,8)(1,2,3)$ as a product of disjoint cycles. This permutation is an element of the group S_9.

14. Let $(G, *)$ be an abelian group. Let S be the set of all elements g in G which satisfy the equation $g * g * g = \text{id}$. Prove that S is a subgroup of G.

15. Let $(G, *)$ be the group $(\mathbb{Z}_3 \times \mathbb{Z}_6, +)$. **LIST** all of the elements of $(G, *)$ which satisfy the equation $g * g * g = \text{id}$. No explanation is needed.

16. Is $(\mathbb{Z}_{15}^{\times}, \times)$ a cyclic group? Explain.

17. Is $(\mathbb{Z}_2 \times \mathbb{Z}_3, +)$ a cyclic group? Explain.

18. The group D_4 has three distinct subgroups of order 4. List the elements of each of these subgroups. (I do not need to see any details.)

19. The subgroup $V = \{(1), (1,2)(3,4), (1,3)(2,4), (1,4)(2,3)\}$ of the group S_4 is normal. (You do not have to prove this.) Find an element of the factor group $\frac{S_4}{V}$ which has order 3. Explain.

20. Let $\mathbb{R}^{\text{pos}}$ represent the group of positive real numbers under multiplication and let U be the unit circle. If z is the complex number $a + b\imath$, then the modulus $|z|$ of z is equal to $\sqrt{a^2 + b^2}$. Define $\varphi\colon \frac{\mathbb{C}\setminus\{0\}}{U} \to \mathbb{R}^{\text{pos}}$ by $\varphi(Uz) = |z|$, for each coset Uz of $\frac{\mathbb{C}\setminus\{0\}}{U}$. Prove that φ is a group isomorphism.