Math 546, Exam 4, Summer, 2002

PRINT Your Name:________________________________

There are 10 problems on 5 pages. Each problem is worth 5 points.

Neither your exam, nor your score, will not be available until class on Monday.

1. Define "group isomorphism". Use complete sentences.

2. Let d be the greatest common divisor of the integers a and b. Prove that there exist integers r and s with $d = ra + sb$.

3. Let G be the subgroup of $(\mathbb{Z}, +)$ which consists of all multiples of 3. Consider the function $\varphi\colon \mathbb{Z} \to G$ which is given by $\varphi(n) = 3n$ for all integers n. Prove that φ is an isomorphism.

4. Prove that the groups $(\mathbb{Z}_4, +)$ and $(\mathbb{Z}_8^{\times}, \times)$ are not isomorphic. The proof does not have to be long, but it does have to be clear.

5. Recall that each element of S_4 is a function from $\{1,2,3,4\}$ to $\{1,2,3,4\}$. Let
$$T = \{\sigma \in S_4 \mid \sigma(1) = 1\}.$$
Is T a subgroup of S_4 ? Prove your answer.

6. Recall that each element of S_4 is a function from $\{1,2,3,4\}$ to $\{1,2,3,4\}$. Let
$$W = \{\sigma \in S_4 \mid \sigma(1) \text{ is equal to either } 1 \text{ or } 2\}.$$
Is W a subgroup of S_4 ? Prove your answer.

7. How many elements of S_4 have order 2 ?

8. Let $\mathbb{R}^{\text{pos}}$ represent the group of positive real numbers under multiplication. Exhibit an isomorphism from the group $\mathbb{R}^{\text{pos}} \times U$ to the group $(\mathbb{C} \setminus \{0\}, \times)$. Prove that your isomorphism really is an isomorphism.

9. Is the group $D_4 \times U_3$ isomorphic to the group S_4 ? Exhibit an isomorphism or prove that the groups are not isomorphic.

10. Express the permutation $(6,9)(4,7,9)(4,8)$ as a product of disjoint cycles.