PRINT Your Name:_____

**Quiz for November 1, 2004**

Consider the function $\varphi\colon \mathrm{GL}_2(\mathbb{R}) \to (\mathbb{R} \setminus \{0\}, \times)$, which is defined by

$$\varphi\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ab.$$

Is $\varphi$ a group homomorphism? Explain thoroughly.

**ANSWER:** The function $\varphi$ is NOT a homomorphism. Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. We see that $A \in \mathrm{GL}_2(\mathbb{R})$. We also see that

$$\varphi(A^2) = \varphi\left(\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\right) = 2.$$

On the other hand, $\varphi(A)\varphi(A) = 1(1) = 1$. Thus, $\varphi(A)\varphi(A) \neq \varphi(A^2)$.